

# Variable Modified Chaplygin Gas and Accelerating Universe

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In this letter, I have proposed a model of variable modified Chaplygin gas and shown its role in accelerating phase of the universe. I have shown that the equation of state of this model is valid from the radiation era to quiescence model. The graphical representations of statefinder parameters characterize different phase of evolution of the universe. All results presented in the letter concerns the case  $k = 0$ .

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Recent observations of the luminosity of type Ia supernovae indicate [1, 2] an accelerated expansion of the universe and lead to the search for a new type of matter which violate the strong energy condition  $\rho + 3p < 0$ . The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as *dark energy*. There are different candidates to play the role of the dark energy. The type of dark energy represented by a scalar field is often called *Quintessence*. In particular one can try another type of dark energy - the so-called *pure Chaplygin gas* which obeys an equation of state like [3]

$$p = -B/\rho, \quad (1)$$

where  $p$  and  $\rho$  are respectively the pressure and energy density and  $B$  is a positive constant. Subsequently the above equation was modified to the form (known as *generalized Chaplygin gas*)

$$p = -B/\rho^\alpha \quad \text{with } 0 \leq \alpha \leq 1. \quad (2)$$

This generalized model has been studied previously [4, 5]. There are some works on *modified Chaplygin gas* obeying an equation of state [6, 7]

$$p = A\rho - \frac{B}{\rho^\alpha} \quad \text{with } 0 \leq \alpha \leq 1, A, B \text{ are positive constants.} \quad (3)$$

This equation of state shows radiation era (when  $A = 1/3$ ) at one extreme (when the scale factor  $a(t)$  is vanishingly small) while a  $\Lambda$ CDM model at the other extreme (when the scale factor  $a(t)$  is infinitely large). Guo and Jhang [8] first proposed *variable Chaplygin gas* model with equation of state (1), where  $B$  is a positive function of the cosmological scale factor 'a' i.e.,  $B = B(a)$ . This assumption is reasonable since  $B(a)$  is related to the scalar potential if we take the Chaplygin gas as a Born-Infeld scalar field [9]. Later there are some works on variable Chaplygin gas model [10].

The metric of a homogeneous and isotropic universe in FRW model is

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (4)$$

where  $a(t)$  is the scale factor and  $k$  ( $= 0, \pm 1$ ) is the constant curvature of their spatial sections.

The Einstein field equations are

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3}\rho \quad (5)$$

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and

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) \quad (6)$$

where  $\rho$  and  $p$  are energy density and isotropic pressure respectively (choosing  $8\pi G = c = 1$ ).

The energy conservation equation is

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad (7)$$

Now, I have introduced *variable modified Chaplygin gas* with equation of state (3) where  $B$  is a positive function of the cosmological scale factor ‘ $a$ ’ (i.e.,  $B = B(a)$ ) as

$$p = A\rho - \frac{B(a)}{\rho^\alpha} \quad \text{with} \quad 0 \leq \alpha \leq 1, \quad A \text{ is constant} > 0. \quad (8)$$

At all stages it shows a mixture. Also in between there is also one stage when the pressure vanishes and the matter content is equivalent to a pure dust.

Now, for simplicity, assume  $B(a)$  is of the form

$$B(a) = B_0 a^{-n} \quad (9)$$

where  $B_0 > 0$  and  $n$  are constants. Using equations (7), (8) and (9), I have the solution of  $\rho$  as

$$\rho = \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A) - n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \quad (10)$$

where  $C > 0$  is an arbitrary integration constant and  $3(1+A)(1+\alpha) > n$ , for positivity of first term. Here  $n$  must be positive, because otherwise,  $a \rightarrow \infty$  implies  $\rho \rightarrow \infty$ , which is not the case for expanding Universe.

From equation (5), for  $k = 0$ , we get the explicit form of  $t$  in terms of  $a$  as

$$t = K a^{\frac{n}{2(1+\alpha)}} {}_2F_1\left[\frac{1}{2(1+\alpha)}, -z, 1-z, -\frac{C}{K} a^{-\frac{n}{2(1+\alpha)z}}\right]$$

where

$$K = \frac{2}{n} \left[ (1+\alpha)^\alpha \sqrt{\frac{n}{6B_0 z}} \right]^{\frac{1}{1+\alpha}}, \quad z = \frac{n}{2(1+\alpha)\{3(1+A)(1+\alpha) - n\}}$$

The deceleration parameter  $q$  has the expression

$$q = -\frac{\ddot{a}}{aH^2}$$

For accelerating universe,  $q$  must be negative i.e.,  $\ddot{a} > 0$  i.e.,

$$\frac{2(1+\alpha) - n}{3(1+\alpha)(1+A) - n} a^{3(1+\alpha)(1+A) - n} > \frac{C(1+3A)}{3B_0} \quad (11)$$

which requires  $n < 2(1+\alpha)$ . Since  $0 \leq \alpha \leq 1$ , this implies  $n \leq 4$ .

This expression shows that for small value of scale factor we have decelerating universe while for large values of scale factor we have accelerating universe and the transition occurs when the scale factor has

the expression  $a = \left[ \frac{C(1+3A)\{3(1+\alpha)(1+A)-n\}}{3B_0\{2(1+\alpha)-n\}} \right]^{\frac{1}{3(1+\alpha)(1+A)-n}}$ .

Now for small value of scale factor  $a(t)$ , I have

$$\rho \simeq \frac{C^{\frac{1}{1+\alpha}}}{a^{3(1+A)}}, \quad (12)$$

which is very large and corresponds to the universe dominated by an equation of state  $p = A\rho$ .

Also for large value of scale factor  $a(t)$ ,

$$\rho \simeq \left( \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \right)^{\frac{1}{1+\alpha}} a^{-\frac{n}{1+\alpha}} \quad (13)$$

and I obtain

$$p = \left( -1 + \frac{n}{3(1+\alpha)} \right) \rho \quad (14)$$

which correspond to *quiescence* model (i.e., dark energy with constant equation of state) [13].

Note that  $n = 0$  corresponds to the original modified Chaplygin gas scenario [7], in which the modified Chaplygin gas behaves initially radiation and later as a cosmological constant. However, equation (10) shows that, in the variable modified Chaplygin gas scenario, it interpolates between a radiation dominated phase ( $A = 1/3$ ) and a quiescence-dominated phase described by the constant equation of state  $p = \gamma\rho$  where  $\gamma = -1 + \frac{n}{3(1+\alpha)} < -\frac{1}{3}$ .

I have described this particular cosmological model from the field theoretical point of view by introducing a scalar field  $\phi$  and a self-interacting potential  $V(\phi)$  with the effective Lagrangian

$$\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (15)$$

In the paper of Gorini et al [4], it has been shown that for the simple flat Friedmann model with Chaplygin gas can equivalently described in terms of a homogeneous minimally coupled scalar field  $\phi$ . Following Barrow [11], Kamenshchik et al [3, 12] have obtained homogeneous scalar field  $\phi(t)$  and a potential  $V(\phi)$  to describe Chaplygin cosmology.

Now, I consider the energy density and pressure corresponding to a scalar field  $\phi$  having a self-interacting potential  $V(\phi)$ . The analogous energy density  $\rho_\phi$  and pressure  $p_\phi$  for the scalar field are the following:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \quad (16)$$

and

$$\begin{aligned} p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = A\rho - \frac{B_0}{\rho^\alpha} = A \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} \\ - B_0 a^{-n} \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{-\frac{\alpha}{1+\alpha}} \end{aligned} \quad (17)$$

Hence for flat universe (i.e.,  $k = 0$ ), I have

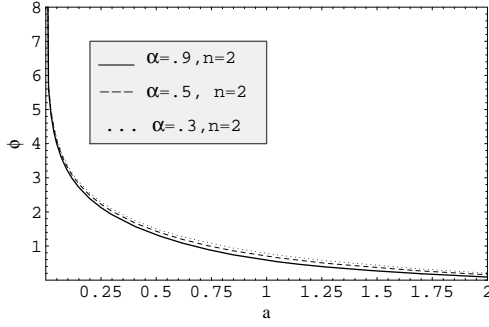


Fig.1

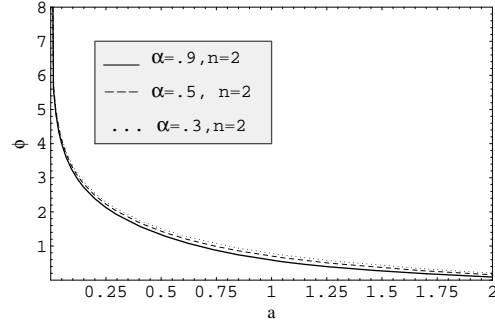


Fig.2

Fig. 1 shows variation of  $\phi$  against  $a$  for  $\alpha = 0.6$  and various values of  $n$  ( $= 1, 2, 3$ ). Fig. 2 shows variation of  $\phi$  against  $a$  for  $n = 2$  and various values of  $\alpha$  ( $= 0.9, 0.5, 0.3$ ).

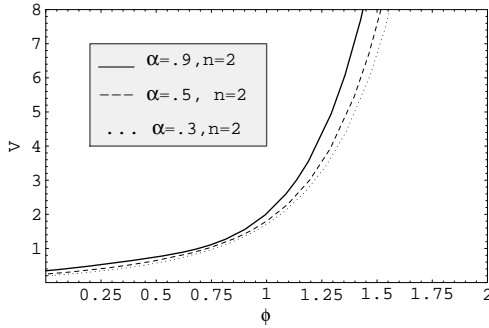


Fig.3

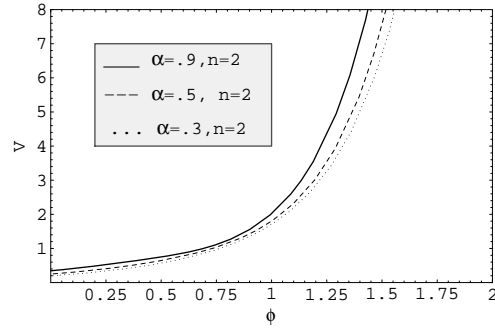


Fig.4

Fig. 3 shows variation of  $V(\phi)$  against  $\phi$  for  $\alpha = 0.6$  and various values of  $n$  ( $= 1, 2, 3$ ). Fig. 4 shows variation of  $V(\phi)$  against  $\phi$  for  $n = 2$  and various values of  $\alpha$  ( $= 0.9, 0.5, 0.3$ ).

$$\phi = \frac{\sqrt{1+A}}{\{3(1+\alpha)(1+A)-n\}} \left[ 2 \log(\sqrt{u+x} + \sqrt{u+y}) - \frac{\sqrt{x}}{\sqrt{y}} \log \left( \frac{(\sqrt{x(u+y)} + \sqrt{y(u+x)})^2}{x^{3/2} \sqrt{y} u} \right) \right] \quad (18)$$

and

$$V(\phi) = \frac{1}{2}(1-A) \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{\frac{1}{1+\alpha}} + \frac{1}{2}B_0a^{-n} \left[ \frac{3(1+\alpha)B_0}{\{3(1+\alpha)(1+A)-n\}} \frac{1}{a^n} + \frac{C}{a^{3(1+A)(1+\alpha)}} \right]^{-\frac{\alpha}{1+\alpha}} \quad (19)$$

where  $x = \frac{n}{1+A}$ ,  $y = 3(1+\alpha)$  and  $u = \frac{nC}{B_0} \left( \frac{y}{x} - 1 \right) a^{n(1-\frac{y}{x})}$ .

The graphical representations of  $\phi$  against  $a$  and  $V(\phi)$  against  $\phi$  have been shown in figures 1, 2 and figures 3, 4 respectively for  $A = 1/3$ . Figures 1 and 3 show the fixed value of  $\alpha = 0.6$  and various values of  $n$  ( $= 1, 2, 3$ ). In this case,  $V(\phi)$  increases with  $\phi$  and slope of the curves decreases as  $n$  increases. Figures 2 and 4 show the fixed value of  $n = 2$  and various values of  $\alpha$  ( $= 0.9, 0.5, 0.3$ ). In this case also,  $V(\phi)$  increases with  $\phi$  and slope of the curves decreases as  $\alpha$  decreases. Figures 1 and 2 describe the

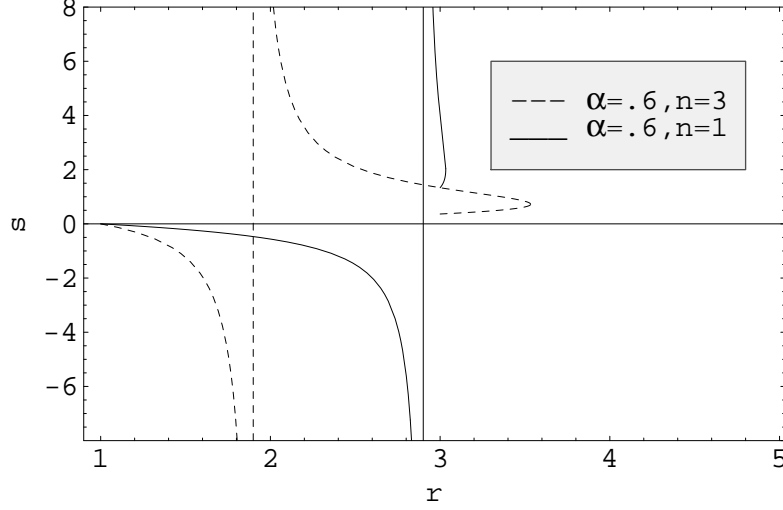


Fig.5

Fig. 5 shows the variation of  $s$  against  $r$  for different values of  $n$  ( $= 3, 1$ ) and for  $\alpha = 0.6$ ,  $A = 1/3$ .

scalar field  $\phi$  always decreases with the evolution of the universe.

In the paper [4], the flat Friedmann model filled with Chaplygin fluid has been analyzed in terms of the recently proposed “*statefinder*” parameters [14]. The statefinder diagnostic along with future SNAP observations may perhaps be used to discriminate between different dark energy models. The above statefinder diagnostic pair i.e.,  $\{r, s\}$  parameters are constructed from the scale factor  $a(t)$  and its derivatives upto the third order as follows:

$$r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \quad (20)$$

where  $H$  is the Hubble parameter and  $q$  ( $= -\frac{a\ddot{a}}{\dot{a}^2}$ ) is the deceleration parameter. These parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The statefinder is dimensionless and is constructed from the scale factor of the Universe and its time derivatives only. The parameter  $r$  forms the next step in the hierarchy of geometrical cosmological parameters after  $H$  and  $q$ .

For Friedmann model with flat universe (i.e.,  $k = 0$ ),

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{1}{3}\rho \quad (21)$$

and

$$q = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3p}{2\rho} \quad (22)$$

So from equation (20) we get

$$r = 1 + \frac{9}{2} \left(1 + \frac{p}{\rho}\right) \frac{\partial p}{\partial \rho}, \quad s = \left(1 + \frac{\rho}{p}\right) \frac{\partial p}{\partial \rho} \quad (23)$$

Thus, I get the ratio between  $p$  and  $\rho$ :

$$\frac{p}{\rho} = \frac{2(r-1)}{9s} \quad (24)$$

For variable modified Chaplygin gas, using equations (7), (23) and (24), I get the relation between  $r$  and  $s$ :

$$18(r-1)s^2 + 18\alpha s(r-1) + 4\alpha(r-1)^2 = 9sA(1+\alpha)(2r+9s-2) + 3ns(2r-2-9sA) \quad (25)$$

Figure 5 shows the variation of  $s$  with the variation of  $r$  for  $A = 1/3$  and for  $\alpha = 0.6$  and  $n = 3, 1$ . The portion of the curve on the positive side of  $s$  which is physically admissible is only the values of  $r$  greater than  $\{1 + \frac{9}{2}A(1+A)\}$ . The part of the curve between  $r = 1$  and  $r = 1 + \frac{9}{2}A(1+A)$  with positive value of  $s$  is not admissible (we have not shown that part in the figure 5) because for the Chaplygin gas under consideration we face a situation, where the magnitude of the constant  $B_0$  becomes negative.

Thus the curve in the positive side of  $s$  starts from radiation era and goes asymptotically to the dust model. But the portion in the negative side of  $s$  represents the evolution from dust state ( $s = -\infty$ ) to the quiescence model. Thus the total curve represents the evolution of the universe starting from the radiation era to the quiescence model.

In this work, I have presented a model for variable modified Chaplygin gas. In this model, I am able to describe the universe from the radiation era ( $A = 1/3$  and  $\rho$  is very large) to quiescence model ( $\rho$  is small constant). So compare to Chaplygin gas model, the present model describe universe to a large extent. Also if, I put  $A = 0$  with  $\alpha = 1$ , then I can recover the results of the Chaplygin gas model. If put  $n = 0$  then variable modified Chaplygin gas model reduces to only modified Chaplygin gas model [7]. In figure 5, for  $\{r, s\}$  diagram the portion of the curve for  $s > 0$  between  $r = 1$  to  $r = 1 + \frac{9}{2}A(1+A)$  is not describable by the modified Chaplygin gas under consideration. For example, if I choose  $r = 1.03$ ,  $A = 1/3$  then from the curve  $s = 0.01$  which corresponds to  $q = 3/2$  and hence I have from the equation of state,  $B < 0$  which is not valid for the specific Chaplygin gas model considered here. At the large value of the scale factor I must have some stage where the pressure becomes negative and hence  $B$  has to be chosen positive. It follows therefore that a portion of the curve as mentioned above should not remain valid.

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### References:

- [1] N. A. Bachall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, *Science* **284** 1481 (1999).
- [2] S. J. Perlmutter et al, *Astrophys. J.* **517** 565 (1999).
- [3] A. Kamenshchik, U. Moschella and V. Pasquier, *Phys. Lett. B* **511** 265 (2001); V. Gorini, A. Kamenshchik, U. Moschella and V. Pasquier, *gr-qc/0403062*.
- [4] V. Gorini, A. Kamenshchik and U. Moschella, *Phys. Rev. D* **67** 063509 (2003); U. Alam, V. Sahni, T. D. Saini and A.A. Starobinsky, *Mon. Not. Roy. Astron. Soc.* **344**, 1057 (2003).
- [5] M. C. Bento, O. Bertolami and A. A. Sen, *Phys. Rev. D* **66** 043507 (2002).
- [6] H. B. Benaoum, *hep-th/0205140*.
- [7] U. Debnath, A. Banerjee and S. Chakraborty, *Class. Quantum Grav.* **21** 5609 (2004).
- [8] Z. K. Guo and Y. Z. Zhang, *Phys. Lett. B* **645** 326 (2007); *astro-ph/0506091*.
- [9] M.C. Bento, O. Bertolami and A.A. Sen, *Phys. Lett. B* **575** 172 (2003).
- [10] G. Sethi, S. K. Singh, P. Kumar, D. Jain and A. Dev, *Int. J. Mod. Phys. D* **15** 1089 (2006); *astro-ph/0508491*; Z. K. Guo and Y. Z. Zhang, *astro-ph/0509790*.
- [11] J. D. Barrow, *Nucl. Phys. B* **310** 743 (1988); *Phys.Lett. B* **235** 40 (1990).
- [12] V. Gorini, A. Yu. Kamenshchik, U. Moschella, V. Pasquier, *Phys.Rev. D* **69** 123512 (2004).
- [13] Z.K. Guo, N. Ohta and Y.Z. Zhang, *astro-ph/0505253*.

- [14] V. Sahni, T. D. Saini, A. A. Starobinsky and U. Alam, *JETP Lett.* **77** 201 (2003).